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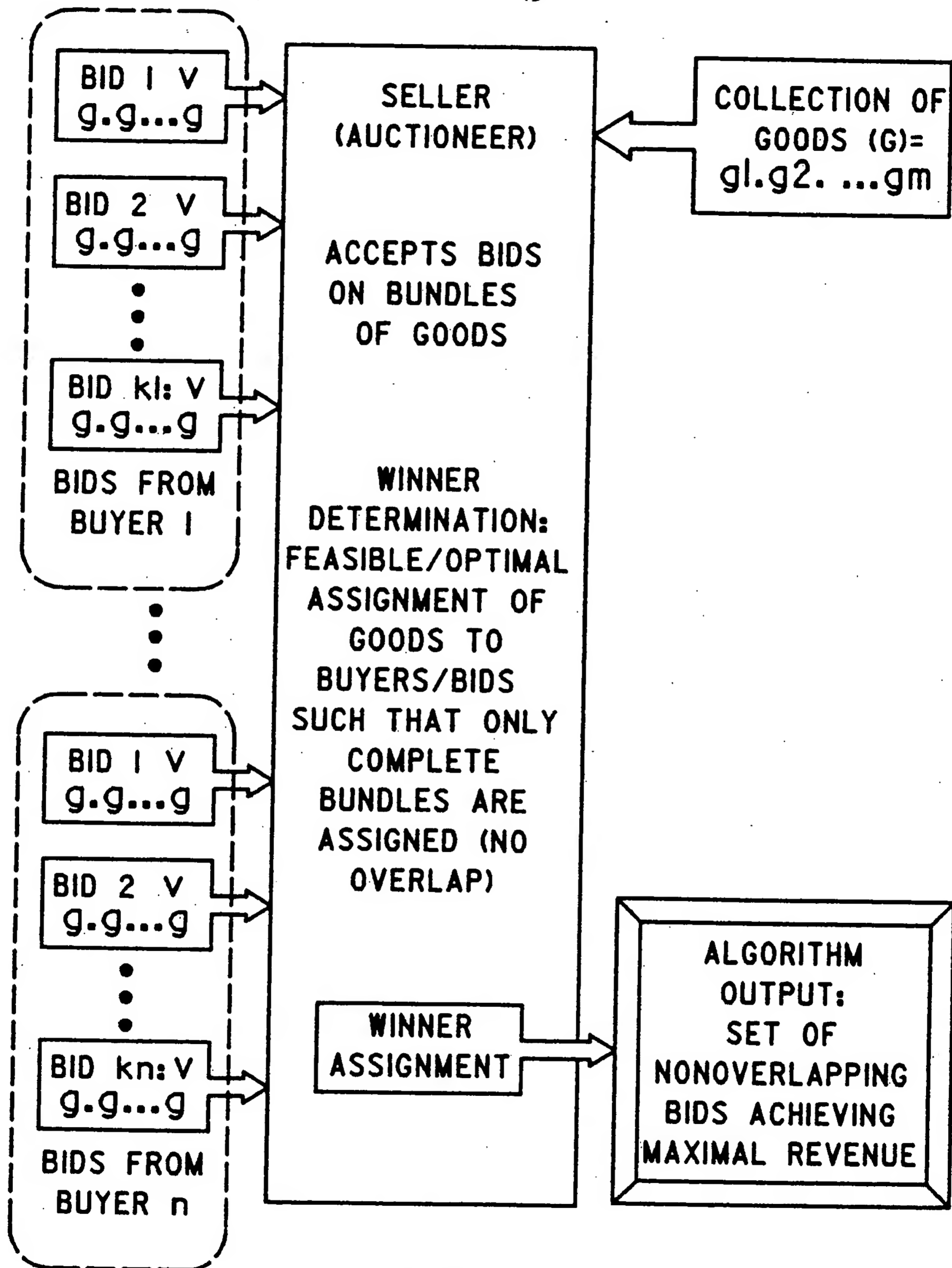


FIG. 1
(PRIOR ART)

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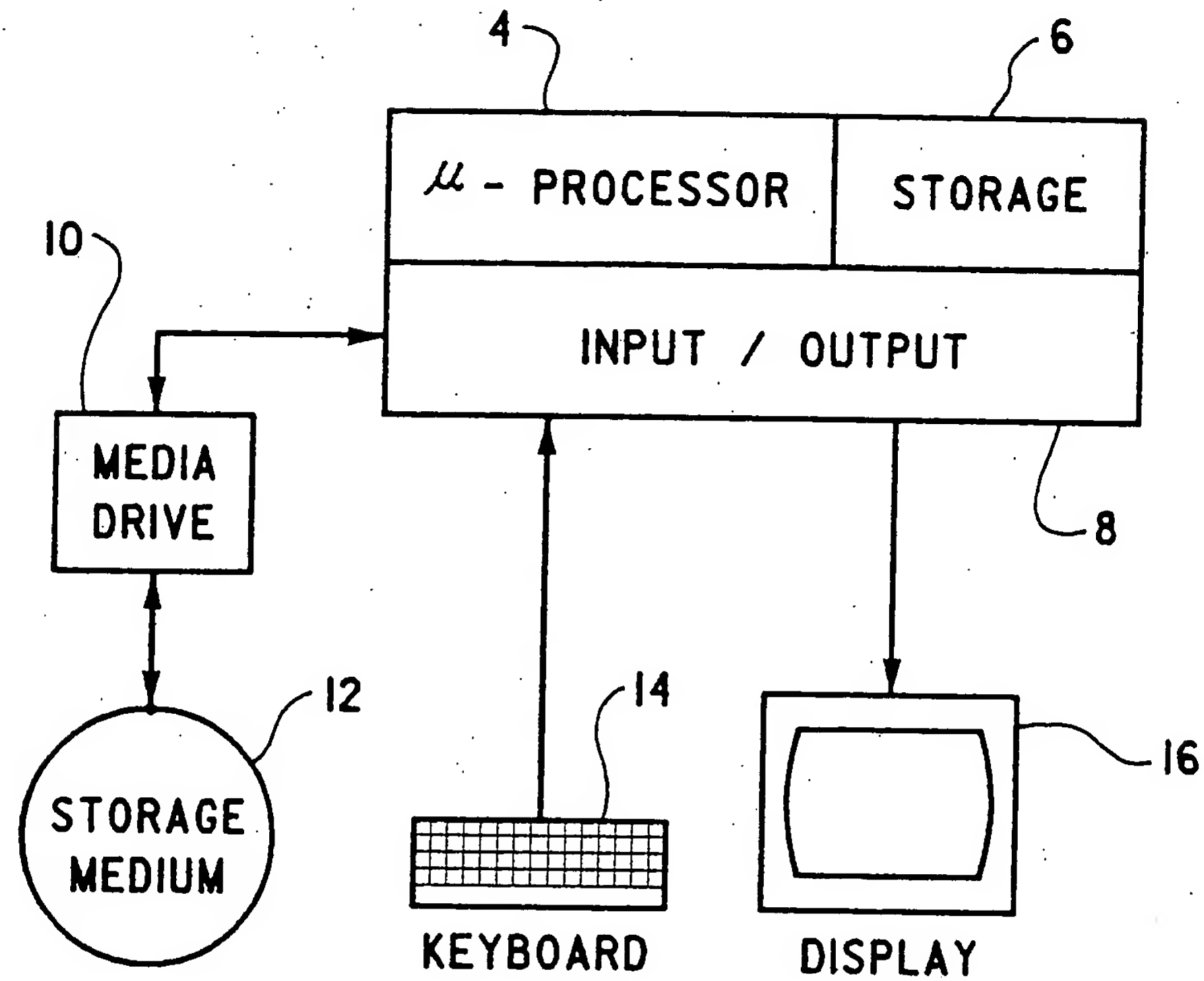
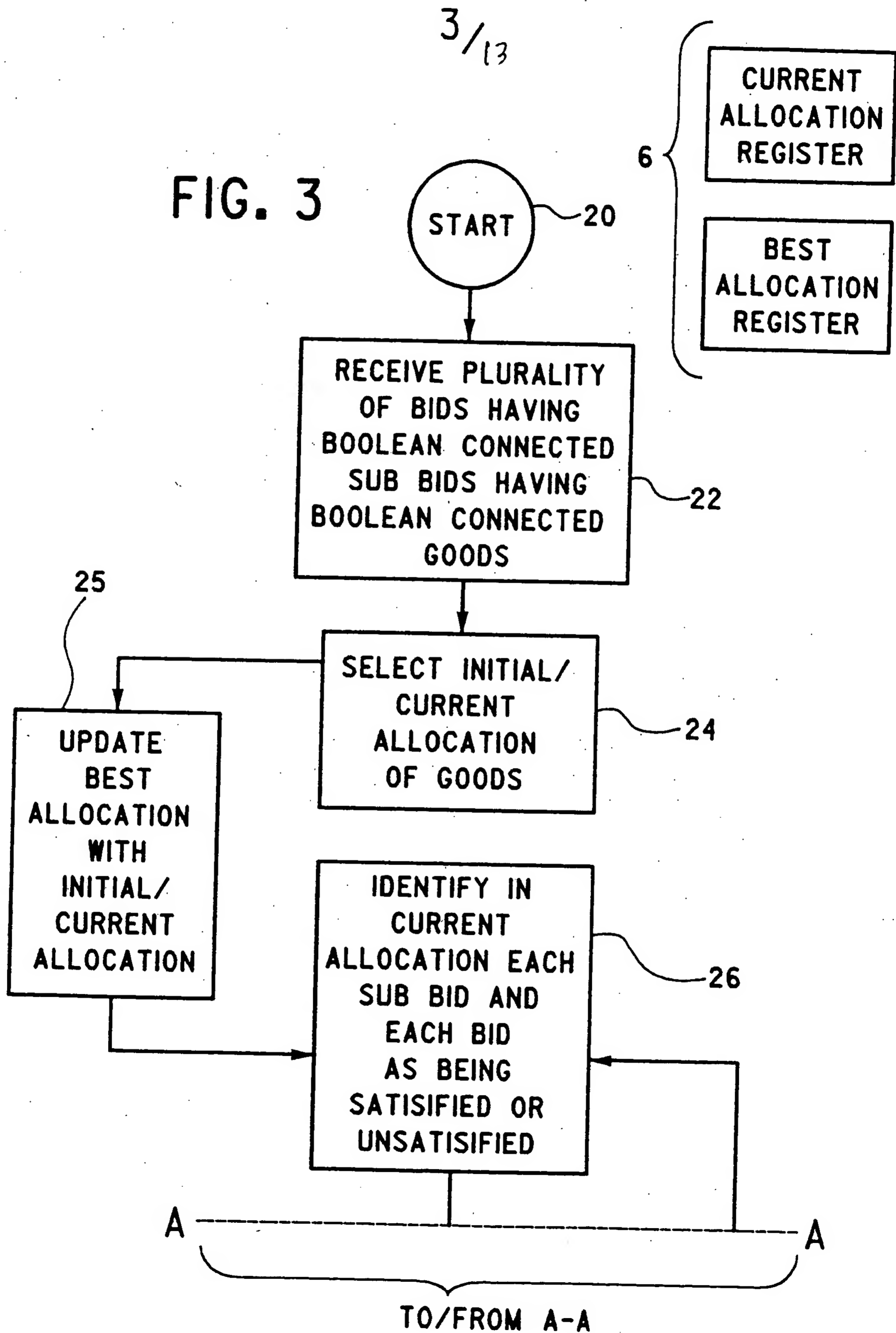
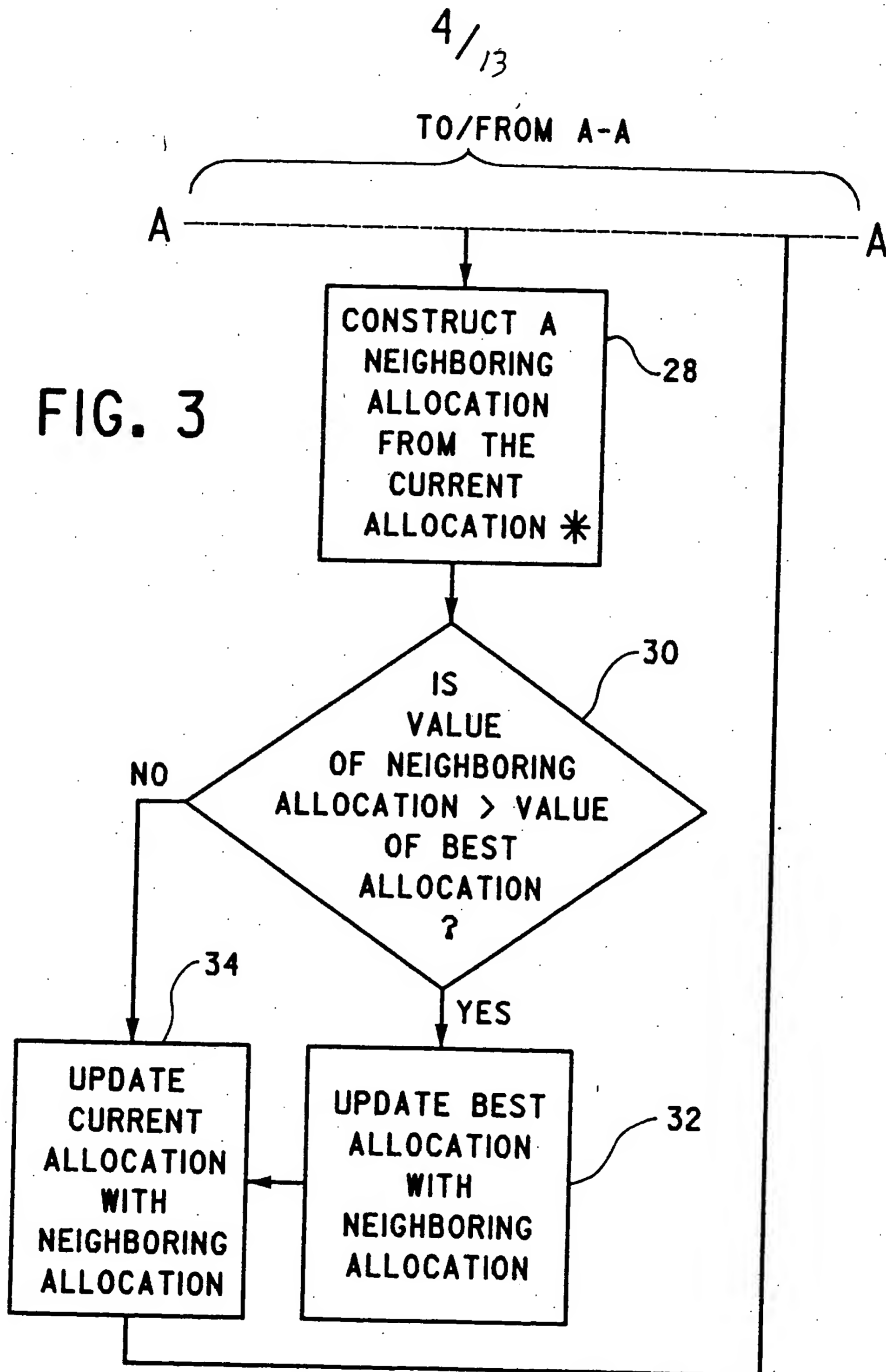


FIG. 2





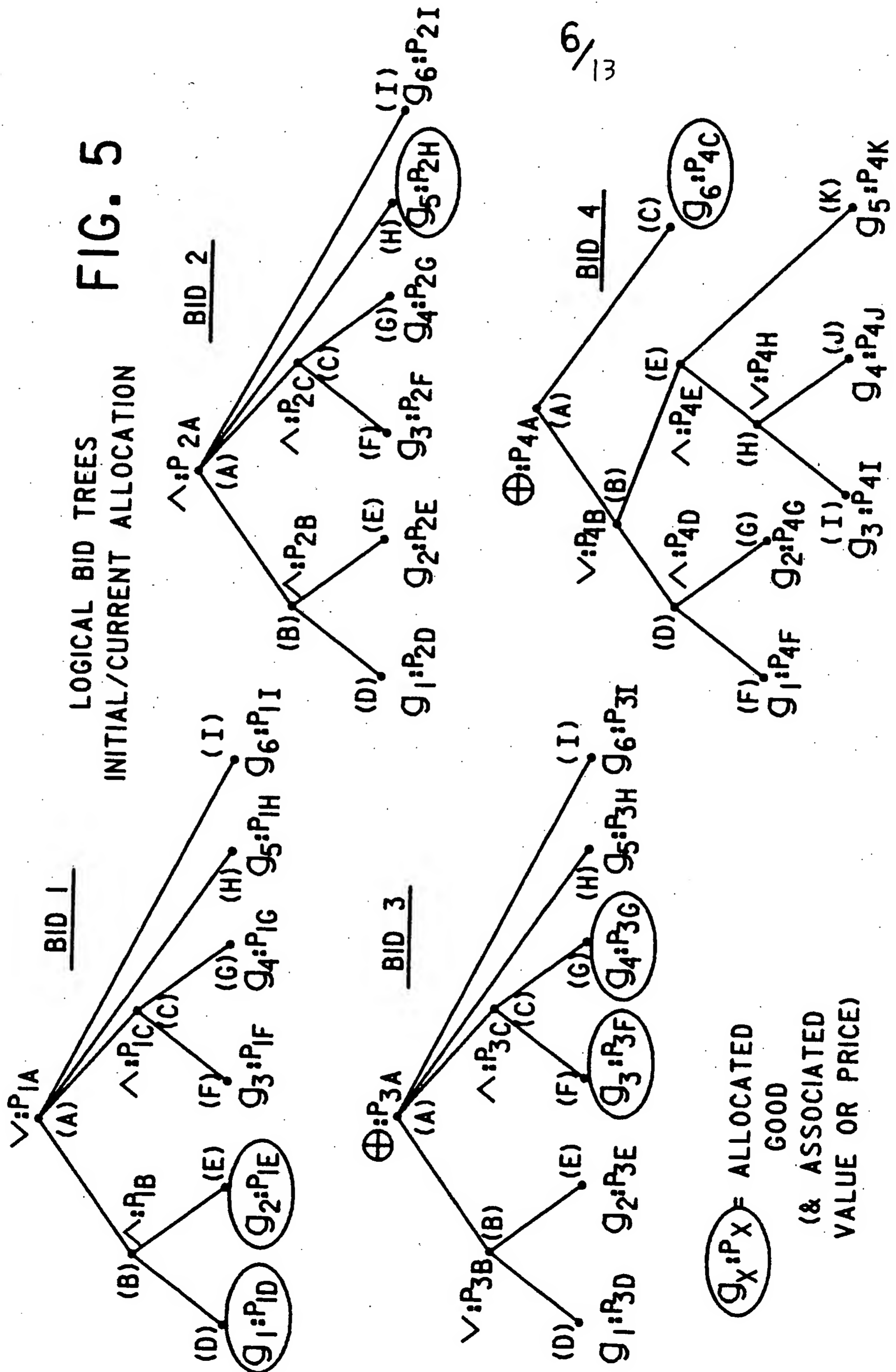
* REALLOCATE AT LEAST ONE GOOD FROM AT LEAST ONE OF THE SUB BIDS OF AT LEAST ONE BID TO ONE OF THE SUB BIDS OF ANOTHER BID.

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$$\begin{aligned}
 \text{BID 1} &= \left[\left(\left(\left(\left(G_1 : P_{1D} \wedge G_2 : P_{1E} \right) P_{1B} \right) \vee \left(G_3 : P_{1F} \wedge G_4 : P_{1G} \right) P_{1C} \right) \vee \left(G_5 : P_{1H} \right) \vee \left(G_6 : P_{1I} \right) \right) P_{1A} \right] \\
 \text{BID 2} &= \left[\left(\left(G_1 : P_{2D} \wedge G_2 : P_{2E} \right) P_{2B} \right) \wedge \left(\left(G_3 : P_{2F} \wedge G_4 : P_{2G} \right) P_{2C} \right) \wedge \left(G_5 : P_{2H} \right) \wedge \left(G_6 : P_{2I} \right) \right] P_{2A} \\
 \text{BID 3} &= \left[\left(\left(\left(G_1 : P_{3D} \vee G_2 : P_{3E} \right) P_{3B} \right) \oplus \left(\left(G_3 : P_{3F} \wedge G_4 : P_{3G} \right) P_{3C} \right) \oplus \left(G_5 : P_{3H} \right) \oplus \left(G_6 : P_{3I} \right) \right) P_{3A} \right] \\
 \text{BID 4} &= \left[\left(\left(\left(\left(\left(G_1 : P_{4F} \wedge G_2 : P_{4G} \right) P_{4D} \right) \vee \left(\left(\left(G_3 : P_{4I} \vee G_4 : P_{4J} \right) P_{4E} \right) \wedge \left(\left(G_5 : P_{4K} \right) P_{4E} \right) \right) \oplus \right. \right. \right. \\
 &\quad \left. \left. \left. \left(G_6 : P_{4C} \right) P_{4A} \right) \right]
 \end{aligned}$$

WHERE \wedge = AND, \vee = OR and \oplus = XOR;
 G = GOOD; and
 P = PRICE or VALUE.

FIG. 4



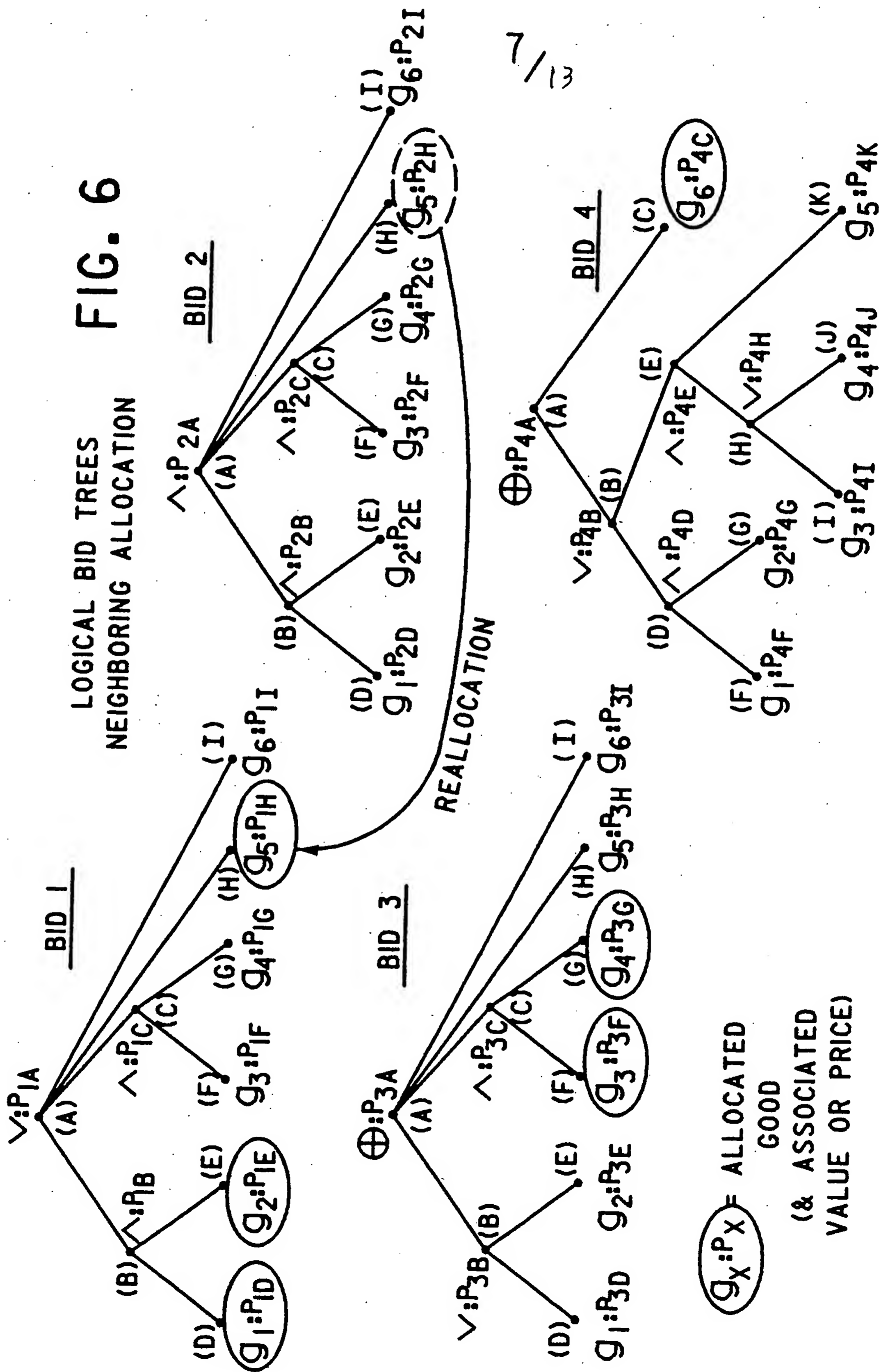
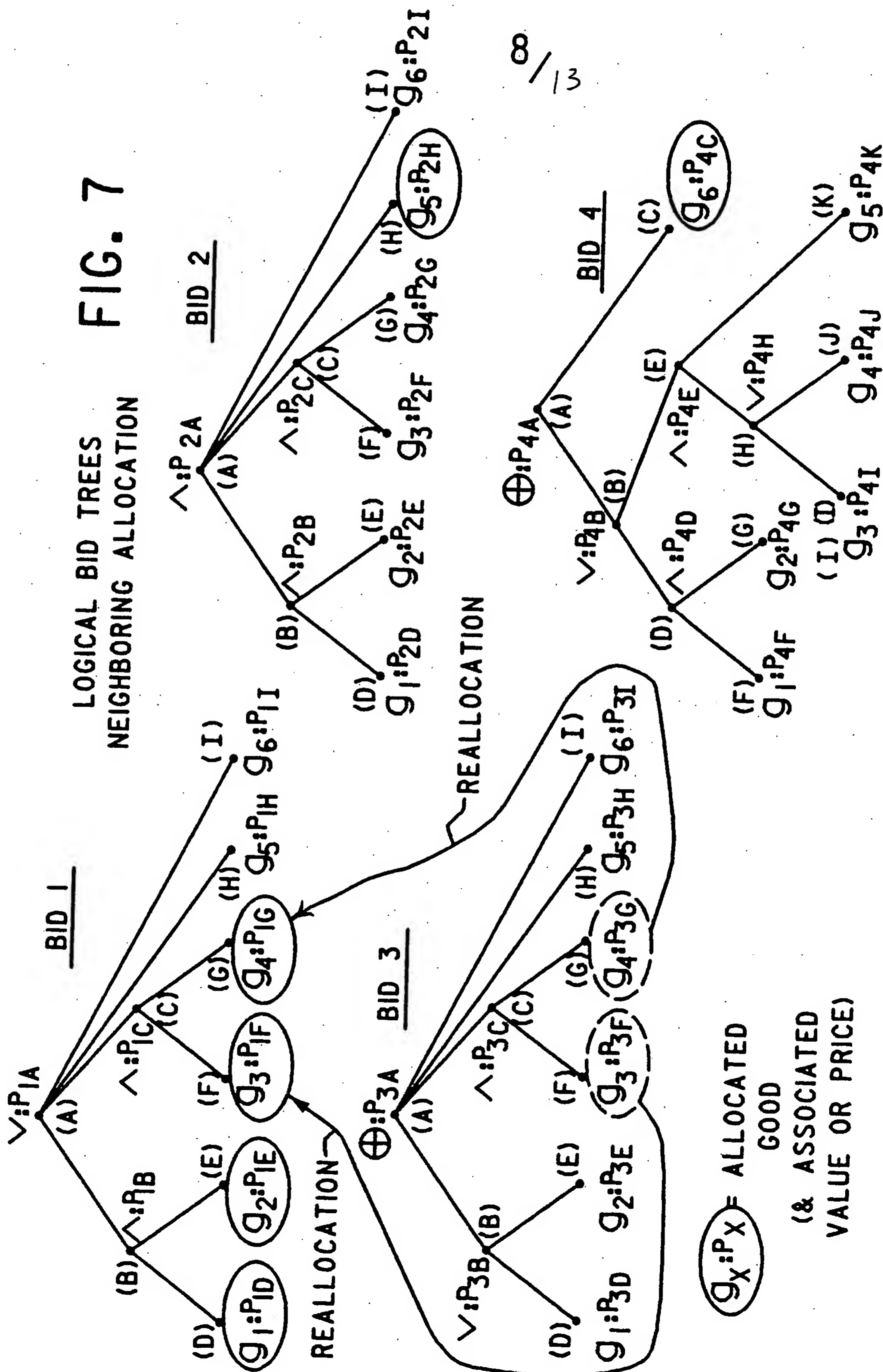


FIG. 7

LOGICAL BID TREES NEIGHBORING ALLOCATION



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 "METHOD AND APPARATUS FOR SOLVING CONCISELY EXPRESSED COMBINATORIAL AUCTION PROBLEMS"

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Fig 8

$$\text{BID 1} = \left[\left(\underbrace{(G_1: P_{1D} \wedge G_2: P_{1E}) P_{1B}}_{60} \right) \vee \left(\underbrace{(G_3: P_{1F} \wedge G_4: P_{1G}) P_{1C}}_{66} \right) \vee \left(\underbrace{(G_5: P_{1H}) \vee (G_6: P_{1I})}_{72} \right) P_{1A} \right]_{76}$$

Variables for Bid 1:

— $X_{11}, X_{12}, X_{13}, X_{14}, X_{15}$ and X_{16} ; where each x is Boolean, i.e.,
 Bid# Good#
 true(1) if good is allocated,
 otherwise false(\emptyset).

— $S_{60}, S_{62}, S_{64}, S_{66}, S_{68}, S_{70}, S_{72}, S_{74}$ and S_{76} ; where each S is Boolean,
 Sub Bid Ref. #
 i.e., true(1) if corresponding sub bid
 is satisfied, otherwise
 false(\emptyset).

— $V_{60}, V_{62}, V_{64}, V_{66}, V_{68}, V_{70}, V_{72}, V_{74}$ and V_{76} ; where each V is the value
 Sub Bid Ref. #
 of the corresponding sub bid
 e.g. V_{60} = value of sub bid
 60 (Integer or Real)

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Fig 9

$$\text{BID 3} = \left[\underbrace{\left(\underbrace{G_1, P_{3D}}_{80} \vee \underbrace{G_2, P_{3E}}_{82} \right) P_{3B}}_{84} \oplus \underbrace{\left(\underbrace{G_3, P_{3F}}_{86} \wedge \underbrace{G_4, P_{3G}}_{88} \right) P_{3C}}_{90} \oplus \underbrace{\left(\underbrace{G_5, P_{3H}}_{92} \oplus \underbrace{G_6, P_{3I}}_{94} \right) P_{3A}}_{96}$$

Variables for Bid 3^o

- $X_{31}, X_{32}, X_{33}, X_{34}, X_{35}, X_{36}$; where each X is Boolean, i.e., true(1) if good is allocated, otherwise false(0).
 Bid # Good #
- $S_{80}, S_{82}, S_{84}, S_{86}, S_{88}, S_{90}, S_{92}, S_{94}, S_{96}$; where each S is Boolean, i.e., true(1) if corresponding sub Bid is satisfied, otherwise false(0).
 Sub Bid Ref #
- $V_{80}, V_{82}, V_{84}, V_{86}, V_{88}, V_{90}, V_{92}, V_{94}, V_{96}$; where each V is the value of the corresponding sub Bid e.g., V_{80} = value of sub Bid 80 (Real or Integer).
 Sub Bid Ref #
- $Z_{84}, Z_{90}, Z_{92}, Z_{94}$; where each Z is Boolean, i.e., true(1) if immediate sub Bid of Reference sub Bid contributes value thereto, otherwise false(0), e.g., Z_{84} is true(1) if good 1, or Z_{92} is allocated to sub Bid 80 or 82, respectively.
 Sub Bid Ref #

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Fig 10

$$B_{ids} = \left[k - \text{of} \left(\overbrace{(g_1 : P_{sa})}^{100}, \overbrace{(g_2 : P_{sc})}^{102}, \overbrace{(g_3 : P_{sd})}^{104} \right) P_{sa} \right]^{106}$$

where k is a real value ≤ 2 .

Variables for B_{ids} :

- X_{s1} , X_{s2} and X_{s3} ; where each x is Boolean true if good is allocated, otherwise Boolean False.
 \uparrow Bid# \uparrow Good#
- S_{100} , S_{102} , S_{104} and S_{106} ; where each x is Boolean true if corresponding sub Bid satisfied, otherwise Boolean False.
 $\underbrace{\quad}_{\text{sub Bid Ref \#}}$
- V_{100} , V_{102} , V_{104} and V_{106} ; where each v is the value of the corresponding sub Bid.
 $\underbrace{\quad}_{\text{sub Bid Ref \#}}$
- n_{106} ; an integer or real value related to the number of satisfied sub bids of Bid s

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FIG. 11(a)

(Atomic):

	<u>Equation 1</u>	<u>Equation 2</u>
Bid 1:	$S_{60} \leq X_{11}$ $S_{62} \leq X_{12}$ $S_{66} \leq X_{13}$ $S_{68} \leq X_{14}$ $S_{72} \leq X_{15}$ $S_{74} \leq X_{16}$	$V_{60} \leq p_{1D} * S_{60}$ $V_{62} \leq p_{1E} * S_{62}$ $V_{66} \leq p_{1F} * S_{66}$ $V_{68} \leq p_{1G} * S_{68}$ $V_{72} \leq p_{1H} * S_{72}$ $V_{74} \leq p_{1I} * S_{74}$
Bid 3:	$S_{80} \leq X_{31}$ $S_{82} \leq X_{32}$ $S_{86} \leq X_{33}$ $S_{88} \leq X_{34}$ $S_{92} \leq X_{35}$ $S_{94} \leq X_{36}$	$V_{80} \leq p_{3D} * S_{80}$ $V_{82} \leq p_{3E} * S_{82}$ $V_{86} \leq p_{3F} * S_{86}$ $V_{88} \leq p_{3G} * S_{88}$ $V_{92} \leq p_{3H} * S_{92}$ $V_{94} \leq p_{3I} * S_{94}$
Bid 5:	$S_{100} \leq X_{51}$ $S_{102} \leq X_{52}$ $S_{104} \leq X_{53}$	$V_{100} \leq p_{5B} * S_{100}$ $V_{102} \leq p_{5C} * S_{102}$ $V_{104} \leq p_{5D} * S_{104}$

FIG. 11(b)

(AND):

	<u>Equation 3</u>	<u>Equation 4</u>
Bid 1:	$2 * S_{64} \leq S_{60} + S_{62}$ $2 * S_{70} \leq S_{66} + S_{68}$	$V_{64} \leq p_{1B} * S_{64} + V_{60} + V_{62}$ $V_{64} \leq p_{1C} * S_{70} + V_{66} + V_{68}$
Bid 3:	$2 * S_{90} \leq S_{86} + S_{88}$	$V_{90} \leq p_{3C} * S_{90} + V_{86} + V_{88}$

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FIG. 11(c)

(OR and XOR):

Equation 5

Equation 6

Bid 1:	$S_{76} \leq S_{64} + S_{70} + S_{72} + S_{74}$	$V_{76} \leq P_{1A} * S_{76} + V_{64} + V_{70} + V_{72} + V_{74}$
Bid 3:	$S_{84} \leq S_{80} + S_{82}$	$V_{84} \leq P_{3B} * S_{84} + V_{80} + V_{82}$
	$S_{96} \leq S_{84} + S_{90} + S_{92} + S_{94}$	$V_{96} \leq P_{3A} * S_{96} + V_{84} + V_{90} + V_{92} + V_{94}$

FIG. 11(d)

(XOR only):

Equation 7

Equation 8

Bid 3:	$t_{84} + t_{90} + t_{92} + t_{94} \leq 1$	$V_{84} \leq \text{MAXVAL} * t_{84}$ $V_{90} \leq \text{MAXVAL} * t_{90}$ $V_{92} \leq \text{MAXVAL} * t_{92}$ $V_{94} \leq \text{MAXVAL} * t_{94}$
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FIG. 11(e)

(k-of) where k = 2:

Bid 5:	<u>Equation 9:</u> $n_{106} \leq S_{100} + S_{102} + S_{104}$	<u>Equation 10:</u> $S_{106} * 2 \leq n_{106}$
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Equation 11:

$$V_{106} \leq P_{5A} * S_{106} + V_{100} + V_{102} + V_{104}$$